

2.4 p 141 • 81, 83, 91, 95, 98, 99, 102

2.5 p 149 5*, 7, 9, 11, 13, 15, 17, 19, 21, 23*, 25, 27, 31, 33*, 35, 37, 39, 41, 43, 45, 47, 49, 51, 53, 55, 57, 59, 61, 63, 65, 67, 69, 71, 73, 75, 77, 79, 81, 83, 85, 87, 89, 91, 93, 95, 97, 99, 101, 103, 105, 107, 109, 111, 113, 115, 117, 119, 121, 123, 125, 127, 129, 131, 133, 135, 137, 139, 141, 143, 145, 147, 149, 151, 153, 155, 157, 159, 161, 163, 165, 167, 169, 171, 173, 175, 177, 179, 181, 183, 185, 187, 189, 191, 193, 195, 197, 199, 201, 203, 205, 207, 209, 211, 213, 215, 217, 219, 221, 223, 225, 227, 229, 231, 233, 235, 237, 239, 241, 243, 245, 247, 249, 251, 253, 255, 257, 259, 261, 263, 265, 267, 269, 271, 273, 275, 277, 279, 281, 283, 285, 287, 289, 291, 293, 295, 297, 299, 301, 303, 305, 307, 309, 311, 313, 315, 317, 319, 321, 323, 325, 327, 329, 331, 333, 335, 337, 339, 341, 343, 345, 347, 349, 351, 353, 355, 357, 359, 361, 363, 365, 367, 369, 371, 373, 375, 377, 379, 381, 383, 385, 387, 389, 391, 393, 395, 397, 399, 401, 403, 405, 407, 409, 411, 413, 415, 417, 419, 421, 423, 425, 427, 429, 431, 433, 435, 437, 439, 441, 443, 445, 447, 449, 451, 453, 455, 457, 459, 461, 463, 465, 467, 469, 471, 473, 475, 477, 479, 481, 483, 485, 487, 489, 491, 493, 495, 497, 499, 501, 503, 505, 507, 509, 511, 513, 515, 517, 519, 521, 523, 525, 527, 529, 531, 533, 535, 537, 539, 541, 543, 545, 547, 549, 551, 553, 555, 557, 559, 561, 563, 565, 567, 569, 571, 573, 575, 577, 579, 581, 583, 585, 587, 589, 591, 593, 595, 597, 599, 601, 603, 605, 607, 609, 611, 613, 615, 617, 619, 621, 623, 625, 627, 629, 631, 633, 635, 637, 639, 641, 643, 645, 647, 649, 651, 653, 655, 657, 659, 661, 663, 665, 667, 669, 671, 673, 675, 677, 679, 681, 683, 685, 687, 689, 691, 693, 695, 697, 699, 701, 703, 705, 707, 709, 711, 713, 715, 717, 719, 721, 723, 725, 727, 729, 731, 733, 735, 737, 739, 741, 743, 745, 747, 749, 751, 753, 755, 757, 759, 761, 763, 765, 767, 769, 771, 773, 775, 777, 779, 781, 783, 785, 787, 789, 791, 793, 795, 797, 799, 801, 803, 805, 807, 809, 811, 813, 815, 817, 819, 821, 823, 825, 827, 829, 831, 833, 835, 837, 839, 841, 843, 845, 847, 849, 851, 853, 855, 857, 859, 861, 863, 865, 867, 869, 871, 873, 875, 877, 879, 881, 883, 885, 887, 889, 891, 893, 895, 897, 899, 901, 903, 905, 907, 909, 911, 913, 915, 917, 919, 921, 923, 925, 927, 929, 931, 933, 935, 937, 939, 941, 943, 945, 947, 949, 951, 953, 955, 957, 959, 961, 963, 965, 967, 969, 971, 973, 975, 977, 979, 981, 983, 985, 987, 989, 991, 993, 995, 997, 999

2.4

81 Horizontal tangent of $f(x) = 2\cos x + \sin 2x$ on $(0, 2\pi)$

$$f'(x) = -2\sin x + 2\cos 2x = 0$$

$$-\sin x + \cos^2 x - \sin^2 x = 0$$

$$-\sin x + (1 - \sin^2 x) - \sin^2 x = 0$$

$$-2\sin^2 x - \sin x + 1 = 0$$

$$2\sin^2 x + \sin x - 1 = 0$$

$$(2\sin x - 1)(\sin x + 1) = 0$$

$$\text{IF } 2\sin x - 1 = 0 \quad \text{IF } \sin x + 1 = 0$$

$$\sin x = \frac{1}{2}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$\sin x = -1$$

$$x = \frac{3\pi}{2}$$

points are

$$\left(\frac{\pi}{6}, \frac{2\sqrt{3}}{3} + \frac{\sqrt{3}}{2}\right), \left(\frac{5\pi}{6}, \frac{2\sqrt{3}}{2} + \frac{\sqrt{3}}{2}\right) \text{ and } \left(\frac{3\pi}{2}, 0\right)$$

$$82) f(x) = 5(2-7x)^4$$

$$f'(x) = 20(2-7x)^3(-7)$$

$$f''(x) = -7 \cdot 20 \cdot 3(2-7x)^2(-7) = 2940(2-7x)^2$$

$$91) f(x) = \cos x^2 \text{ find } f''(0) \text{ at } (0, 1)$$

$$f'(x) = (-\sin x^2)(2x) = -2x \sin x^2$$

$$f''(x) = -2x(\cos x^2(2x)) + \sin x^2(-2)$$

$$f''(0) = -4x^2 \cos x^2 - 2\sin x^2 \Big|_{x=0} = 0$$

$$95) (a) g(x) = f(3x)$$

$$g'(x) = f'(3x) \cdot 3$$

so g 's slope is 3 times greater than f

$$(b) g(x) = f(x^2)$$

$$g'(x) = f'(x^2)(2x)$$

g 's rate is $2x$ times faster

$$98) \left. \begin{array}{l} g(5) = -3 \\ g'(5) = 6 \end{array} \right\} \left. \begin{array}{l} h(5) = 3 \\ h'(5) = -2 \end{array} \right\} \text{ find } f'(5)$$

$$(a) f'(5) = (g(x)h(x))' \Big|_{x=5} = g(5)h'(5) + g'(5)h(5) = (-3)(-2) + (6)(3) = 24$$

(98) (b) $f(5) = g(h(5)) = g(3)$
 $f'(x) = g'(h(x)) h'(x) \Rightarrow f'(5) = g'(3) h'(5) = ? (-2)$

(c) $f(x) = \frac{g(x)}{h(x)}$

$f'(x) = \frac{h(x)g'(x) - h'(x)g(x)}{[h(x)]^2} = \frac{(3)(6) - (-2)(-3)}{9} = \frac{12 - 6}{9} = \frac{6}{9} = \frac{2}{3}$

(d) $f(x) = [g(x)]^3$

$f'(x) = 3[g(x)]^2 g'(x) = 3(-3)^2(6) = 162$

(99) Use graph to find values & slopes

$h(x) = f(g(x))$

$g(x) = g(f(x))$

(a) $h'(x) = f'(g(x)) \cdot g'(x)$ find $h'(1)$

$= f'(g(1)) \cdot g'(1) = f'(4) \cdot (-\frac{1}{2}) = -1 \cdot \frac{1}{2} = -\frac{1}{2}$

(b) $g'(x) = g'(f(x)) \cdot f'(x)$ find $g'(5)$

$= g'(f(5)) \cdot f'(5) = g'(6) \cdot (-1) = \text{DNE} (-1)$

(102)

$y = \frac{1}{3} \cos 12t - \frac{1}{4} \sin 12t$

(y = Feet
t = seconds)

velocity: $y' = -4 \sin 12t - 3 \cos 12t$

pos is $\frac{1}{3} \cos 12(\frac{\pi}{8}) - \frac{1}{4} \sin 12(\frac{\pi}{8}) = 0 - \frac{1}{4}(-1) = \frac{1}{4}$ foot

vel is $-4 \sin 12(\frac{\pi}{8}) - 3 \cos 12(\frac{\pi}{8})$

$-4(\sin \frac{3\pi}{2}) - 3 \cos(\frac{3\pi}{2})$

$-4(-1) - 0$

4 feet per second

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Find $\frac{dy}{dx}$ implicitly

(5) $x^2 + y^2 = 9$
 $2x \frac{dx}{dx} + 2y \frac{dy}{dx} = 0$

$$\frac{dy}{dx} = \frac{-2x}{2y} = -\frac{x}{y}$$

(7) $x^5 + y^5 = 16$
 $5x^4 \frac{dx}{dx} + 5y^4 \frac{dy}{dx} = 0$

$$\frac{dy}{dx} = \frac{-5x^4}{5y^4} = -\frac{x^4}{y^4}$$

(9) $x^3 - xy + y^2 = 7$ ← product rule

$$3x^2 \left(\frac{dx}{dx} \right) - \left[x \frac{dy}{dx} + y \frac{dx}{dx} \right] + 2y \frac{dy}{dx} = 0$$

$$-x \frac{dy}{dx} + 2y \frac{dy}{dx} = -3x^2 + y$$

$$\frac{dy}{dx} (-x + 2y) = -3x^2 + y$$

$$\frac{dy}{dx} = \frac{-3x^2 + y}{-x + 2y}$$

(11) $x^3 y^3 - y = x$

$$x^3 \left(3y^2 \frac{dy}{dx} \right) + 3x^2 \left(\frac{dx}{dx} \right) y^3 - 1 \left(\frac{dy}{dx} \right) = 1 \left(\frac{dx}{dx} \right)$$

$$3x^3 y^2 \left(\frac{dy}{dx} \right) - \left(\frac{dy}{dx} \right) = 1 - 3x^2 y^3$$

$$\left(\frac{dy}{dx} \right) (3x^3 y^2 - 1) = 1 - 3x^2 y^3$$

$$\frac{dy}{dx} = \frac{1 - 3x^2 y^3}{3x^3 y^2 - 1}$$

(13) $x^3 - 3x^2 y + 2xy^2 = 12$

$$3x^2 \left(\frac{dx}{dx} \right) - \left[3x^2 \frac{dy}{dx} + 6xy \right] + \left[2x \cdot 2y \frac{dy}{dx} + 2y^2 \right] = 0$$

$$-3x^2 \left(\frac{dy}{dx} \right) + 4xy \left(\frac{dy}{dx} \right) = -3x^2 + 6xy - 2y^2$$

$$(-3x^2 + 4xy) \left(\frac{dy}{dx} \right) = -3x^2 + 6xy - 2y^2$$

$$\frac{dy}{dx} = \frac{-3x^2 + 6xy - 2y^2}{-3x^2 + 4xy}$$

HW 1-15 Page 4

2.5 (15) $\sin x + 2 \cos 2y = 1$

$$\cos x \left(\frac{dx}{dx} \right) - 4 \sin 2y \left(\frac{dy}{dx} \right) = 0$$

$$\frac{dy}{dx} = \frac{-\cos x}{-4 \sin 2y} = \frac{\cos x}{4 \sin 2y}$$

(17) $\csc x = x(1 + \tan y)$

$$-(\csc x \cot x) \left(\frac{dx}{dx} \right) = x \sec^2 y \left(\frac{dy}{dx} \right) + (1 + \tan y)$$

$$\frac{-\csc x \cot x - 1 - \tan y}{x \sec^2 y} = \frac{dy}{dx}$$

(19) $y = \sin xy$

$$1 \left(\frac{dy}{dx} \right) = (\cos xy) \left(x \left(\frac{dy}{dx} \right) + y \right)$$

$$\frac{dy}{dx} - x \cos xy \left(\frac{dy}{dx} \right) = y \cos xy$$

$$\frac{dy}{dx} (1 - x \cos xy) = y \cos xy$$

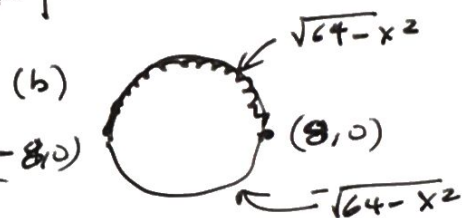
$$\frac{dy}{dx} = \frac{y \cos xy}{1 - x \cos xy}$$

(21) $x^2 + y^2 = 64$ explicitly

(a) $y = \begin{cases} \sqrt{64-x^2} \\ -\sqrt{64-x^2} \end{cases}$

(c) part 1: $y' = \frac{1}{2}(64-x^2)^{-1/2}(-2x)$

part 2: $y' = -\frac{1}{2}(64-x^2)^{-1/2}(-2x)$



(d) $2x + 2y \frac{dy}{dx} = 0$

$$\frac{dy}{dx} = \frac{-2x}{2y} = -\frac{x}{y}$$

The Same? Part 1: $\frac{-x}{\sqrt{64-x^2}}$ Part 2: $\frac{x}{\sqrt{64-x^2}}$

but $y = \pm \sqrt{64-x^2}$ so $= -\frac{x}{y}$

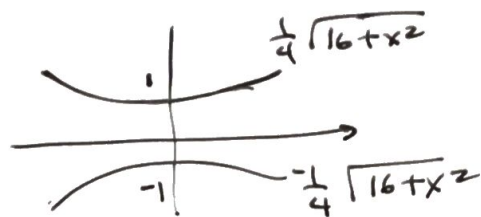
(23)

$$16y^2 - x^2 = 16$$

$$(a) y^2 = \frac{16 + x^2}{16}$$

$$y = \frac{\pm \sqrt{16 + x^2}}{4}$$

(b)



$$(c) y_1' = \frac{1}{4} \cdot \frac{1}{2} (16 + x^2)^{-1/2} (2x)$$

$$= \frac{x}{4\sqrt{16 + x^2}}$$

$$y_2' = -\frac{1}{4} \cdot \frac{1}{2} (16 + x^2)^{-1/2} (2x)$$

$$= \frac{-x}{4\sqrt{16 + x^2}}$$

Same?

(d) Implicitly:

$$32y \left(\frac{dy}{dx} \right) - 2x = 0$$

$$\frac{dy}{dx} = \frac{2x}{32y} = \frac{x}{16y}$$

$$\frac{x}{16 \left(\frac{\pm \sqrt{16 + x^2}}{4} \right)} = \frac{\pm x}{4\sqrt{16 + x^2}} \checkmark$$

(25)

$$xy = 6 \text{ (Product Rule)}$$

$$x \frac{dy}{dx} + (1)y = 0$$

$$\frac{dy}{dx} = -\frac{y}{x}$$



find slope here

$$\frac{-(-1)}{-6} = -\frac{1}{6}$$

(27)

$$y^2 = \frac{x^2 - 49}{x^2 + 49} \text{ @ } (7, 0)$$

$$2y \frac{dy}{dx} = \frac{(x^2 + 49)(2x) - (x^2 - 49)(2x)}{(x^2 + 49)^2}$$

$$\frac{dy}{dx} = \frac{2x [(x^2 + 49) - (x^2 - 49)]}{2y(x^2 + 49)^2} = \frac{x(49 + 49)}{y(x^2 + 49)^2}$$

when $y=0$, undefined slope

$$(31) \tan(x+y) = x \text{ @ } (0, 0)$$

$$\sec^2(x+y) \left[1 + 1 \left(\frac{dy}{dx} \right) \right] = 1$$

$$\sec^2(x+y) + \left(\frac{dy}{dx} \right) (\sec^2(x+y)) = 1$$

$$\frac{dy}{dx} = \frac{1 - \sec^2(x+y)}{\sec^2(x+y)} = \cos^2(x+y) - 1 = -\sin^2(x+y)$$

$$\text{so at } (0, 0) = \sin 0 = 0$$

(33)



$(x^2+4)y = 8$ find slope at $(2,1)$

$$(x^2+4) \frac{dy}{dx} + (2x)y = 0$$

$$\frac{dy}{dx} = \frac{-2xy}{x^2+4} \bigg|_{(2,1)} = \frac{-2(2)(1)}{2^2+4} = \frac{-4}{8} = -\frac{1}{2}$$

(35)



$$(x^2+y^2)^2 = 4x^2y$$

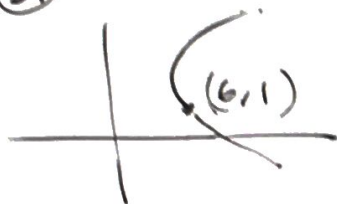
$$2(x^2+y^2)(2x+2y(\frac{dy}{dx})) = 4x^2 \frac{dy}{dx} + (8x)y$$

$$\frac{dy}{dx} (4y(x^2+y^2) - 4x^2) = 8xy - 4x(x^2+y^2)$$

$$\frac{dy}{dx} = \frac{8xy - 4x(x^2+y^2)}{4y(x^2+y^2) - 4x^2}$$

$$\frac{dy}{dx} \bigg|_{(1,1)} = \frac{8 - 8}{8 - 4} = 0$$

(37)

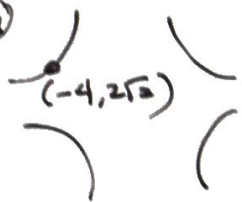


$$(y-3)^2 = 4(x-5)$$

$$2(y-3) \frac{dy}{dx} = 4 \Rightarrow \frac{dy}{dx} = \frac{2}{y-3} \bigg|_{y=1} = -1$$

tangent line $y-1 = -1(x-6)$

(39)



$$x^2y^2 - 9x^2 - 4y^2 = 0$$

$$x^2 2y \left(\frac{dy}{dx} \right) + 2xy^2 - 18x - 8y \left(\frac{dy}{dx} \right) = 0$$

$$\frac{dy}{dx} (2x^2y - 8y) = -2xy^2 + 18x$$

$$\frac{dy}{dx} = \frac{-2xy^2 + 18x}{2x^2y - 8y}$$

$$\text{at } (-4, 2\sqrt{3}) \text{ slope is } \frac{(-2)(-4)(12) + 18(-4)}{2(8)(2\sqrt{3}) - 16\sqrt{3}} = \frac{24}{16\sqrt{3}}$$

$$y - 2\sqrt{3} = \frac{3}{2\sqrt{3}}(x+4)$$

(45) Ellipse

(a)

$$\frac{x^2}{2} + \frac{y^2}{8} = 1 \quad \text{or} \quad 4x^2 + y^2 = 8$$

slope

$$\frac{1}{2}(2x) + \frac{1}{8}(2y) \frac{dy}{dx} = 0 \quad \text{or} \quad 8x + 2y \left(\frac{dy}{dx} \right) = 0$$

$$\frac{dy}{dx} = \frac{-x}{\frac{1}{4}y} = -\frac{4x}{y} \quad \left| \quad \frac{dy}{dx} = \frac{-8x}{2y} = -\frac{4x}{y} \right.$$

tangent

@ (1,2)

$$y - 2 = -\frac{4}{2}(x - 1)$$

$$y = -2x + 4$$

(b) In general

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

is

$$b^2 x^2 + a^2 y^2 = a^2 b^2$$

so slope is

$$2b^2 x + 2a^2 y \left(\frac{dy}{dx} \right) = 0$$

$$\frac{dy}{dx} = \frac{-2b^2 x}{2a^2 y} = -\frac{b^2 x}{a^2 y}$$

tangent line @ (x_0, y_0) is

$$y - y_0 = \frac{-b^2 x}{a^2 y} (x - x_0)$$

$$\frac{a^2 y^2 - a^2 y y_0}{a^2 b^2}$$

$$= \frac{-b^2 x^2 + b^2 x x_0}{a^2 b^2}$$

$$\frac{y^2}{b^2} - \frac{y y_0}{b^2} = -\frac{x^2}{a^2} + \frac{x x_0}{a^2}$$

so this
is one
by given

$$\boxed{\frac{y^2}{b^2} + \frac{x^2}{a^2}} = \frac{x_0 x}{a^2} + \frac{y_0 y}{b^2}$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$