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2.4 p 141 · 81, 83, 91, 95, 98, 99, 102

2.5 p 149 5*, 7, 9, 11, 13, 15, 17, 19, 21, 23*, 25, 27, 31, 33*, 35, 37, 39, 45

2.4

81 Horizontal tangent of $f(x) = 2\cos x + \sin 2x$ on $(0, 2\pi)$

$$f'(x) = -2\sin x + 2\cos 2x = 0$$

$$-\sin x + \cos^2 x - \sin^2 x = 0$$

$$-\sin x + (1 - \sin^2 x) - \sin^2 x = 0$$

$$-2\sin^2 x - \sin x + 1 = 0$$

$$2\sin^2 x + \sin x - 1 = 0$$

$$(2\sin x - 1)(\sin x + 1) = 0$$

If $2\sin x - 1 = 0$ If $\sin x + 1 = 0$

$\sin x = \frac{1}{2}$ $\sin x = -1$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$ $x = \frac{3\pi}{2}$

points are

$\left(\frac{5\pi}{6}, -\frac{2\sqrt{3}}{3} + \frac{\sqrt{3}}{2}\right)$ $\left(\frac{\pi}{6}, \frac{2\sqrt{3}}{2} + \frac{\sqrt{3}}{2}\right)$ and $\left(\frac{3\pi}{2}, 0\right)$

83 $f(x) = 5(2-7x)^4$

$f'(x) = 20(2-7x)^3(-7)$

$f''(x) = -7 \cdot 20 \cdot 3(2-7x)^2(-7) = 2940(2-7x)^2$

91 $f(x) = \cos x^2$ find $f''(0)$ at $(0,1)$

$f'(x) = (-\sin x^2)(2x) = -2x \cdot \sin x^2$

$f''(x) = -2x(\cos x^2(2x)) + \sin x^2(-2)$

$f''(0) = -4x^2\cos x^2 - 2\sin x^2\Big|_{x=0} = 0$

95 (a) $g(x) = f(3x)$

$g'(x) = f'(3x) \cdot 3$

so g's slope is 3 times greater than f

(b) $g(x) = f(x^2)$

$g'(x) = f'(x^2)(2x)$

g's rate is $2x$ times faster

98 $g(5) = -3$ $h(5) = 3$

$g'(5) = 6$ $h'(5) = -2$ find $f'(5)$

(a) $f'(x) = (g(x)h(x))'\Big|_{x=5} = g(5)h'(5) + g'(5)h(5) = (-3)(-2) + (6)(3) = 24$

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9. (b) $f(5) = g(h(5)) = g(3)$

$f'(x) = g'(h(x))h'(x) \Rightarrow f'(5) = g'(3)h'(5) = ?\ (-2)$

(c) $f(5) = \dfrac{g(5)}{h(5)}$

$f'(5) = \dfrac{h(5)g'(5) - h'(5)g(5)}{[h(5)]^2} = \dfrac{(3)(6)-(-2)(-3)}{9} = \dfrac{12}{9} = \dfrac{4}{3}$

(d) $f(5) = [g(5)]^3$

$f'(5) = 3[g(5)]^2 g'(5) = 3(-3)^2(6) = 162$

99. Use graph to find values & slopes

$h(x) = f(g(x))$

$s(x) = g(f(x))$

(a) $h'(x) = f'(g(x)) \cdot g'(x)$ find $h'(1)$

$= f'(g(1)) \cdot g'(1) = f'(4) \cdot (-\frac{1}{2}) = -1 \cdot -\frac{1}{2} = \frac{1}{2}$

(b) $s'(x) = g'(f(x)) \cdot f'(x)$ find $s'(5)$

$g'(f(5)) \cdot f'(5) = g'(6) \cdot (-1) = \text{DNE} \ (-1)$

102. $y = \frac{1}{3}\cos 12t - \frac{1}{4}\sin 12t$ (y = feet, t = seconds)

velocity: $y' = -4\sin 12t - 3\cos 12t$

pos is $\frac{1}{3}\cos 12(\frac{\pi}{8}) - \frac{1}{4}\sin 12(\frac{\pi}{8}) = 0 - \frac{1}{4}(-1) = \frac{1}{4}$ foot

vel is $-4\sin 12(\frac{\pi}{8}) - 3\cos 12t$

$-4(\sin \frac{3\pi}{2}) - 3\cos(\frac{3\pi}{2})$

$-4(-1) - 0$

4 feet per second

$\boxed{25}$ P 149 find $\frac{dy}{dx}$ implicitly

(5.) $x^2 + y^2 = 9$

$2x\frac{dx}{dx} + 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-2x}{2y} = \frac{-x}{y}$

(7.) $x^5 + y^5 = 16$

$5x^4\frac{dx}{dx} + 5y^4\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-5x^4}{5y^4} = \frac{-x^4}{y^4}$

(9)

$x^3 - \overset{\swarrow \text{product rule}}{xy} + y^2 = 7$

$3x^2\left(\frac{dx}{dx}\right) - \left[x\frac{dy}{dx} + y\frac{dx}{dx}\right] + 2y\frac{dy}{dx} = 0$

$-x\frac{dy}{dx} + 2y\frac{dy}{dx} = -3x^2 + y$

$\frac{dy}{dx}(-x + 2y) = -3x^2 + y$

$\frac{dy}{dx} = \frac{-3x^2 + y}{-x + 2y}$

(11) $x^3y^3 - y = x$

$x^3\left(3y^2\frac{dy}{dx}\right) + 3x^2\left(\frac{dx}{dx}\right)y^3 - 1\left(\frac{dy}{dx}\right) = 1\left(\frac{dx}{dx}\right)$

$3x^3y^2\left(\frac{dy}{dx}\right) - \left(\frac{dy}{dx}\right) = 1 - 3x^2y^3$

$\left(\frac{dy}{dx}\right)(3x^3y^2 - 1) = 1 - 3x^2y^3$

$\frac{dy}{dx} = \frac{1 - 3x^2y^3}{3x^3y^2 - 1}$

(13) $x^3 - 3x^2y + 2xy^2 = 12$

$3x^2\left(\frac{dx}{dx}\right) - \left[3x^2\frac{dy}{dx} + 6xy\right] + \left[2x \cdot 2y\frac{dy}{dx} + 2y^2\right] = 0.$

$-3x^2\left(\frac{dy}{dx}\right) + 4xy\left(\frac{dy}{dx}\right) = -3x^2 + 6xy - 2y^2$

$(-3x^2 + 4xy)\left(\frac{dy}{dx}\right) = -3x^2 + 6xy - 2y^2$

$\frac{dy}{dx} = \frac{-3x^2 + 6xy - 2y^2}{-3x^2 + 4xy}$

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$\boxed{2.5}$ (15) $\sin x + 2\cos 2y = 1$

$$\cos x\left(\frac{dx}{dx}\right) - 4\sin 2y\left(\frac{dy}{dx}\right) = 0$$

$$\frac{dy}{dx} = \frac{-\cos x}{-4\sin 2y} = \frac{\cos x}{4\sin 2y}$$

(17) $\csc x = x(1 + \tan y)$

$$-(\csc x \cot x)\left(\frac{dx}{dx}\right) = x\sec^2 y\left(\frac{dy}{dx}\right) + (1+\tan y)$$

$$\frac{-\csc x\cot x - 1 - \tan y}{x\sec^2 y} = \frac{dy}{dx}$$

(19) $y = \sin xy$

$$1\left(\frac{dy}{dx}\right) = (\cos xy)\left(x\left(\frac{dy}{dx}\right) + y\right)$$

$$\frac{dy}{dx} - x\cos xy\left(\frac{dy}{dx}\right) = y\cos xy$$

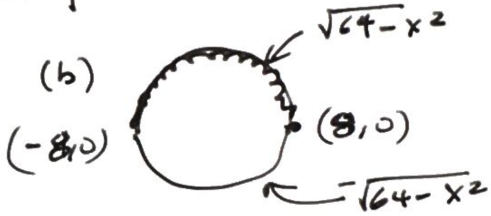
$$\frac{dy}{dx}(1 - x\cos xy) = y\cos xy$$

$$\frac{dy}{dx} = \frac{y\cos xy}{1 - x\cos xy}$$

(21) $x^2 + y^2 = 64$ explicitly

(a) $y = \begin{cases} \sqrt{64-x^2} \\ -\sqrt{64-x^2} \end{cases}$

(b)



(c) part 1: $y' = \frac{1}{2}(64-x^2)^{-1/2}(-2x)$

part 2: $y' = -\frac{1}{2}(64-x^2)^{-1/2}(-2x)$

(d) $2x + 2y\frac{dy}{dx} = 0$

$$\frac{dy}{dx} = \frac{-2x}{2y} = \frac{-x}{y}$$

The Same? Part 1: $\frac{-x}{\sqrt{64-x^2}}$ Part 2: $\frac{x}{\sqrt{64-x^2}}$

but $y = \pm\sqrt{64-x^2}$ so $= \frac{-x}{y}$

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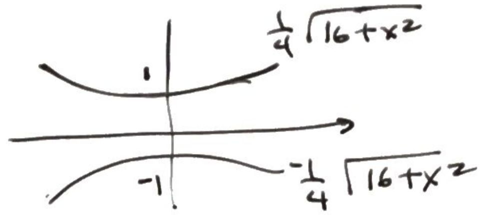
(23) $16y^2 - x^2 = 16$

(a) $y^2 = \dfrac{16+x^2}{16}$

$y = \dfrac{\pm\sqrt{16+x^2}}{4}$

(b)



(c) $y_1' = \frac{1}{4}\cdot\frac{1}{2}(16+x^2)^{-1/2}(2x)$

$= \dfrac{x}{4\sqrt{16+x^2}}$

$y_2' = -\frac{1}{4}\cdot\frac{1}{2}(16+x^2)^{-1/2}(2x)$

$= \dfrac{-x}{4\sqrt{16+x^2}}$

Same?

(d) Implicitly:

$32y\left(\frac{dy}{dx}\right) - 2x = 0$

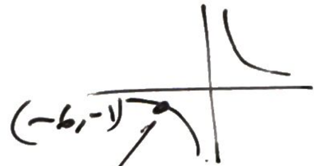
$\dfrac{dy}{dx} = \dfrac{2x}{32y} = \dfrac{x}{16y}$

$\dfrac{x}{16\left(\pm\frac{1}{4}\sqrt{16+x^2}\right)} = \dfrac{\pm x}{4\sqrt{16+x^2}}$ ✓

(25) $xy = 6$ (Product Rule)

$x\dfrac{dy}{dx} + (1)y = 0$

$\dfrac{dy}{dx} = \dfrac{-y}{x}$



find slope here

$\dfrac{-(-1)}{-6} = -1/6$

(27) $y^2 = \dfrac{x^2-49}{x^2+49}$ @ $(7,0)$

$2y\dfrac{dy}{dx} = \dfrac{(x^2+49)(2x) - (x^2-49)(2x)}{(x^2+49)^2}$

$\dfrac{dy}{dx} = \dfrac{2x\left[(x^2+49) - (x^2-49)\right]}{2y(x^2+49)^2} = \dfrac{x(49+49)}{y(x^2+49)^2}$

when $y=0$, undefined slope

(31) $\tan(x+y) = x$ @ $(0,0)$

$\sec^2(x+y)\left[1 + 1\left(\frac{dy}{dx}\right)\right] = 1$

$\sec^2(x+y) + \left(\frac{dy}{dx}\right)(\sec^2(x+y)) = 1$

$\dfrac{dy}{dx} = \dfrac{1-\sec^2(x+y)}{\sec^2(x+y)} = \cos^2(x+y) - 1 = -\sin^2(x+y)$

so at $(0,0) = \sin 0 = 0$

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$(x^2+4)y = 8$ find slope at $(2,1)$

$(x^2+4)\frac{dy}{dx} + (2x)y = 0$

$\frac{dy}{dx} = \left.\frac{-2xy}{x^2+4}\right|_{(2,1)} = \frac{-2(2)(1)}{2^2+4} = \frac{-4}{8} = -\frac{1}{2}$

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$(x^2+y^2)^2 = 4x^2y$

$2(x^2+y^2)\left(2x+2y\left(\frac{dy}{dx}\right)\right) = 4x^2\frac{dy}{dx} + (8x)y$

$\frac{dy}{dx}\left(4y(x^2+y^2) - 4x^2\right) = 8xy - 4x(x^2+y^2)$

$\frac{dy}{dx} = \frac{8xy - 4x(x^2+y^2)}{4y(x^2+y^2) - 4x^2}$

$\left.\frac{dy}{dx}\right|_{(1,1)} = \frac{8-8}{8-4} = 0$

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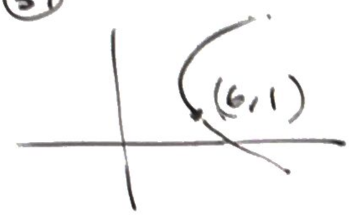

$(y-3)^2 = 4(x-5)$

$2(y-3)\frac{dy}{dx} = 4 \Rightarrow \frac{dy}{dx} = \left.\frac{2}{y-3}\right|_{y=1} = -1$

tangent line $y-1 = -1(x-6)$

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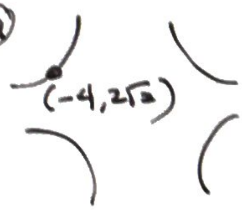

$x^2y^2 - 9x^2 - 4y^2 = 0$

$x^2 2y\left(\frac{dy}{dx}\right) + 2xy^2 - 18x - 8y\left(\frac{dy}{dx}\right) = 0$

$\frac{dy}{dx}\left(2x^2y - 8y\right) = -2xy^2 + 18x$

$\frac{dy}{dx} = \frac{-2xy^2 + 18x}{2x^2y - 8y}$

@ $(-4, 2\sqrt{3})$ slope is $\frac{(-2)(-4)(12) + 18(-4)}{2(8)(2\sqrt{3}) - 16\sqrt{3}} = \frac{24}{16\sqrt{3}}$

$y - 2\sqrt{3} = \frac{3}{2\sqrt{3}}(x+4)$

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(45) Ellipse

(a) $\frac{x^2}{2} + \frac{y^2}{8} = 1$ or $4x^2 + y^2 = 8$

slope $\frac{1}{2}(2x) + \frac{1}{8}(2y)\frac{dy}{dx} = 0$ or $8x + 2y\left(\frac{dy}{dx}\right) = 0$

$\frac{dy}{dx} = \frac{-x}{\frac{1}{4}y} = -\frac{4x}{y}$ ✓ $\quad \frac{dy}{dx} = \frac{-8x}{2y} = \frac{-4x}{y}$

tangent @ (1,2) $y - 2 = -\frac{4}{2}(x-1)$

$y = -2x + 4$

(b) In general $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

is $b^2x^2 + a^2y^2 = a^2b^2$

so slope is

$2b^2x + 2a^2y\left(\frac{dy}{dx}\right) = 0$

$\frac{dy}{dx} = \frac{-2b^2x}{2a^2y} = \frac{-b^2x}{a^2y}$

tangent line @ (x_0, y_0) is

$y - y_0 = \frac{-b^2x}{a^2y}(x - x_0)$

$\frac{a^2y^2 - a^2yy_0}{a^2b^2} = \frac{-b^2x^2 + b^2xx_0}{a^2b^2}$

$\frac{y^2}{b^2} - \frac{yy_0}{b^2} = \frac{-x^2}{a^2} + \frac{xx_0}{a^2}$

$\boxed{\frac{y^2}{b^2} + \frac{x^2}{a^2}} = \frac{x_0x}{a^2} + \frac{y_0y}{b^2}$

so this is one by given $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$